A Robust Method for Characterization of Optical Waveguides and Couplers

Minh A. Tran, Tin Komljenovic, Jared C. Hulme, Michael L. Davenport, and John E. Bowers

***Abstract*— We propose a simple yet powerful method to characterize waveguide propagation loss and 2 × 2 waveguide coupler's coupling coefficient simultaneously. The method, based on the spectrum analysis of transmission through an unbalanced Mach–Zehnder interferometer, requires only a single test structure and is insensitive to variation of input coupling losses. We show that, in most general cases, it is possible to determine both the propagation loss and the coupling of both couplers that can have different coupling coefficients.**

***Index Terms*— Integrated circuit testing, couplers, optical coupling, loss measurement.**

I. Introduction

WAVEGUIDES and couplers are among the most essential building blocks for any photonic integrated circuits (PIC). In the literature, various methods for measuring waveguide propagation loss have been reported. However, most of them require either multiple structures with different lengths (variation of the cut-back method) [1] or a known reflectivity (Fabry-Perot based methods) [2], [3]. A disadvantage of cut-back based methods lies in their sensitivity to coupling efficiency variation among multiple devices, resulting in a relatively large uncertainty or a need to test a larger number of devices. Fabry-Perot loss modulation methods are, as was already pointed out, sensitive to facet reflectivity. The measurement of the coupling ratios of waveguide couplers in general also suffers from the input coupling loss issue, where the unequal losses of the coupling between fibers to waveguides directly leads to inaccuracy of the results.

The aforementioned issues may be eliminated by analyzing optical spectral transmission of micro-ring or Mach-Zehnder Interferometer (MZI) structures [4]–[8]. For ring based structures, curve fitting generally must be used. This requires several device splits to discern the coupling and the waveguide loss or to obtain the coupling regimes of interest [5], [6]. With MZI based structures, methods using an unbalanced MZI (UMZI) formed by a directional coupler and a multi-mode interference coupler [7] or two Y-branches [8] have been proposed. However, existing analysis techniques for these structures can only extract the coupling coefficient of the coupler [7] or still require multiple test structures to extract the waveguide loss [8].

In this letter, we propose a new method for simultaneously extracting both the waveguide propagation loss and the power coupling ratios of the couplers with the use of only one single test structure and without any fitting processes. The test structure is a UMZI formed by two (not necessarily identical) 2 × 2 couplers. We show that by simply extracting the extinction ratios of the UMZI output transmission spectra, it is possible to calculate the coupling ratios of each coupler and the waveguide loss. This measurement method is also insensitive to the fiber-waveguide coupling losses, thus it can achieve precise results without the need for fiber-waveguide coupling repeatability.

The letter is organized as follows. In Section II we introduce the theory behind the characterization method and derive formulas that describe the responses of a general unbalanced MZI. In Section III, two different test cases are performed to illustrate the characterization method in practice.

II. Theory

Our test structure comprises an unbalanced MZI. The UMZI is formed by two 2 × 2 couplers connected to each other by two waveguide arms with different path lengths, as shown in Fig. 1.a. Said 2 × 2 couplers can, but do not have to be identical. Typical optical spectra of transmissions are shown in Fig. 1.b. At a wavelength window of interest (1550 ± 1.5 nm in this example), extinction ratios (ERs) are defined as the ratio between the neighboring maxima and minima of the optical spectra. The goal of this section is to establish explicit formulae for the waveguide propagation loss constant and the coupling coefficients of the two 2 × 2 couplers as a function of said ERs. Let the power coupling and power transmission coefficients of a waveguide directional coupler be denoted by $\kappa^2 = \sin^2\theta_i$ and $\tau^2 = \cos^2\theta_i$ respectively (where $0 \le \theta_i \le \frac{\pi}{2}$). Then, the scattering matrix of the coupler can be written as $C_i = \begin{bmatrix} \cos\theta_i & -j\sin\theta_i \\ -j\sin\theta_i & \cos\theta_i \end{bmatrix}$.

The path length difference between the two arms is ΔL, thus the transfer matrix of the delay path in between the two couplers is $T_i = \begin{bmatrix} \exp(-j\phi - \alpha\Delta L) & 0 \\ 0 & 1 \end{bmatrix}$ where $\phi = \frac{2\pi n_{eff}(\lambda)}{\lambda}\Delta L$ expresses the accumulated phase shift difference between the two arms and $\alpha(\mathrm{cm}^{-1})$ is the field propagation loss constant of the waveguide. Notice that the power

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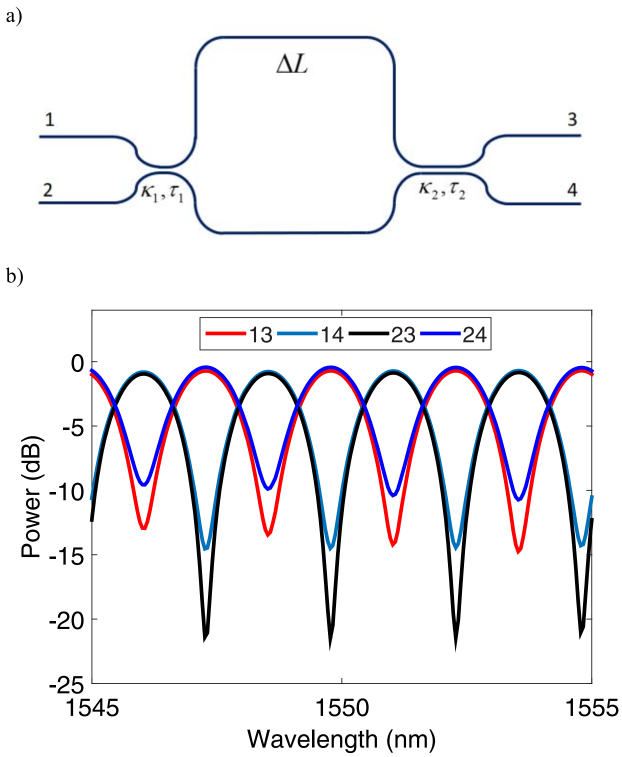

Fig. 1. a) Schematic of an UMZI. b) Optical spectra of transmissions through a UMZI structure. The spectra were simulated with the coupling ratios of the two couplers being 0.3 and 0.4, path length difference $\Delta L = 500\ \mu$m and a propagation loss 10 dB/cm. Extinction ratios are denoted by R_{13}, R_{14}, R_{23} and R_{24}.

propagation loss of the waveguide in dB/cm can be converted from this constant by the formula $\alpha_{dB} = \frac{20}{\ln 10}\alpha$ (dB/cm).

We will first establish the analytical expression for the transmission powers through the UMZI using these parameters. The transfer matrix of the whole MZI structure is given by $S = C_1 T C_2 = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ where

$$S_{11} = \exp(-j\phi - \alpha \Delta L)\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \quad (1.a)$$

$$S_{12} = -j\sin\theta_1 \cos\theta_2 - j\exp(-j\phi - \alpha\Delta L)\cos\theta_1 \sin\theta_2 \quad (1.b)$$

$$S_{21} = -j\exp(-j\phi - \alpha\Delta L)\sin\theta_1 \cos\theta_2 - j\cos\theta_1 \sin\theta_2 \quad (1.c)$$

$$S_{22} = \cos\theta_1 \cos\theta_2 - \exp(-j\phi - \alpha\Delta L)\sin\theta_1 \sin\theta_2 \quad (1.d)$$

Now we can calculate the coupling powers between ports. For instance, the power coupling from port 1 to port 3 of the UMZI is given by $P_{13} = |S_{11}^2|$. The full equation is

$$P_{13} = \exp(-2\alpha\Delta L)\cos^2\theta_1 \cos^2\theta_2 + \sin^2\theta_1 \sin^2\theta_2 + \\ -2\cos\phi \exp(-\alpha\Delta L)\cos\theta_1 \cos\theta_2 \sin\theta_1 \sin\theta_2 \quad (2)$$

Similarly, one can obtain the equations for the coupling powers between all other ports. Due to space considerations we will follow the full notation for P_{13} only, but all other power couplings (P_{14}, P_{23}, P_{24}) are straightforward to derive.

In the scope of this letter, we only consider the regime where the path length difference ΔL is at least two orders of magnitude larger than the wavelength λ. The wavelength period for $\phi = \frac{2\pi n_{eff}(\lambda)}{\lambda}\Delta L$ to complete a full 2π cycle is $\Delta\lambda = \frac{\lambda^2}{\Delta L n_g(\lambda)}$ where $n_g(\lambda) = n_{eff}(\lambda) - \lambda\frac{dn_{eff}(\lambda)}{d\lambda}$. In the considered regime, the wavelength period $\Delta\lambda$ is small enough to allow us to assume that all the terms except $\cos\phi$ in Eq. (1) and Eq. (2) are approximately constant within the wavelength interval $[\lambda - \Delta\lambda/2, \lambda + \Delta\lambda/2]$. Certainly, the longer ΔL is, the more valid the assumption becomes and the more accurate the measurement is.

With that assumption, we will next establish the equations for the extrema of the coupling power within the wavelength interval $[\lambda - \Delta\lambda/2, \lambda + \Delta\lambda/2]$. In Eq. 2, it is clear that the coupling power reaches the extrema when $\cos\phi$ reaches its extrema.

$$P_{13}^{\max,\min} = (\exp(-\alpha\Delta L)\cos\theta_1 \cos\theta_2 \pm \sin\theta_1 \sin\theta_2)^2 \quad (3)$$

From the equation above and defining the ER to be the ratio between the maxima and minima, we have the following equation for the ER of the power transmission spectrum from port 1 to 3:

$$R_{13} = \frac{P_{13}^{\max}}{P_{13}^{\min}} = \left(\frac{\exp(-\alpha\Delta L) + \tan\theta_1 \tan\theta_2}{\exp(-\alpha\Delta L) - \tan\theta_1 \tan\theta_2}\right)^2 \quad (4)$$

These equations relate the ERs, which we can directly measure experimentally, with the propagation loss $\exp(-\alpha\Delta L)$ and the characteristic parameters $\tan\theta_1$ and $\tan\theta_2$ of the two couplers, which are what we want to calculate. In the next step, we will derive the values of α, θ_1 and θ_2 from the directly measurable values of R_{13}, R_{14}, R_{23} and R_{24}.

From the full equation set (derived R_{13}, R_{14}, R_{23} and R_{24}), we obtain the following equations:

$$\frac{\exp(-\alpha\Delta L) + \tan\theta_1 \tan\theta_2}{\exp(-\alpha\Delta L) - \tan\theta_1 \tan\theta_2} = \pm\sqrt{R_{13}} \quad (5.a)$$

$$\frac{\exp(-\alpha\Delta L)\tan\theta_1 + \tan\theta_2}{\exp(-\alpha\Delta L)\tan\theta_1 - \tan\theta_2} = \pm\sqrt{R_{14}} \quad (5.b)$$

$$\frac{\tan\theta_1 + \exp(-\alpha\Delta L)\tan\theta_2}{\tan\theta_1 - \exp(-\alpha\Delta L)\tan\theta_2} = \pm\sqrt{R_{23}} \quad (5.c)$$

$$\frac{1 + \exp(-\alpha\Delta L)\tan\theta_1 \tan\theta_2}{1 - \exp(-\alpha\Delta L)\tan\theta_1 \tan\theta_2} = \pm\sqrt{R_{24}} \quad (5.d)$$

Based on the pair (5.a, 5.d), we can solve for $\exp(-\alpha\Delta L)$ and $\tan\theta_1 \tan\theta_2$.

$$\exp(-2\alpha\Delta L) = \frac{\sqrt{R_{13}} + (-1)^l}{\sqrt{R_{13}} - (-1)^l} \cdot \frac{\sqrt{R_{24}} + (-1)^m}{\sqrt{R_{24}} - (-1)^m} \quad (6.a)$$

$$\tan\theta_1 \tan\theta_2 = \frac{\sqrt{R_{13}} - (-1)^l}{\sqrt{R_{13}} + (-1)^l} \cdot \frac{\sqrt{R_{24}} + (-1)^m}{\sqrt{R_{24}} - (-1)^m} \quad (6.b)$$

Similarly, we have the following solutions for $\exp(-\alpha\Delta L)$ and $\tan\theta_2/\tan\theta_1$ based on the pair (5.b, 5.c):

$$\exp(-2\alpha\Delta L) = \frac{\sqrt{R_{14}} + (-1)^n}{\sqrt{R_{14}} - (-1)^n} \cdot \frac{\sqrt{R_{23}} + (-1)^p}{\sqrt{R_{23}} - (-1)^p} \quad (6.c)$$

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\sqrt{R_{14}} - (-1)^n}{\sqrt{R_{14}} + (-1)^n} \cdot \frac{\sqrt{R_{23}} + (-1)^p}{\sqrt{R_{23}} - (-1)^p} \quad (6.d)$$

Here, the parameters *l, m, n, p* can be either 0 or 1. However, this set of parameters are constrained by the condition that (6.a) and (6.c) must be equivalent and the value of $\exp(-\alpha\Delta L)$ must be smaller than 1, as written in (7) below.

$$\frac{\sqrt{R_{13}}+(-1)^l}{\sqrt{R_{13}}-(-1)^l}\cdot\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m} = \frac{\sqrt{R_{14}}+(-1)^n}{\sqrt{R_{14}}-(-1)^n}\cdot\frac{\sqrt{R_{23}}+(-1)^p}{\sqrt{R_{23}}-(-1)^p} < 1 \quad (7.a)$$

This condition will eliminate all the unsatisfying values of *l, m, n, p* and leave out only one relevant set. Finally, based on equations (6) with the set of *l, m, n, p* determined, the parameters α, κ_1 and κ_2 can be explicitly expressed:

$$\alpha = -\frac{1}{2\Delta L}\ln\left(\frac{\sqrt{R_{14}}+(-1)^n}{\sqrt{R_{14}}-(-1)^n}\frac{\sqrt{R_{23}}+(-1)^p}{\sqrt{R_{23}}-(-1)^p}\right) = -\frac{1}{2\Delta L}\ln\left(\frac{\sqrt{R_{13}}+(-1)^l}{\sqrt{R_{13}}-(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\right) \quad (8.a)$$

$$\kappa_1 = \sin^2\theta_1 = \frac{\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\frac{\sqrt{R_{14}}-(-1)^n}{\sqrt{R_{14}}+(-1)^n}\frac{\sqrt{R_{23}}+(-1)^p}{\sqrt{R_{23}}-(-1)^p}\right)^{1/2}}{1+\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\frac{\sqrt{R_{14}}-(-1)^n}{\sqrt{R_{14}}+(-1)^n}\frac{\sqrt{R_{23}}+(-1)^p}{\sqrt{R_{23}}-(-1)^p}\right)^{1/2}} \quad (8.b)$$

$$\kappa_2 = \sin^2\theta_2 = \frac{\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\frac{\sqrt{R_{14}}+(-1)^n}{\sqrt{R_{14}}-(-1)^n}\frac{\sqrt{R_{23}}-(-1)^p}{\sqrt{R_{23}}+(-1)^p}\right)^{1/2}}{1+\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\frac{\sqrt{R_{14}}+(-1)^n}{\sqrt{R_{14}}-(-1)^n}\frac{\sqrt{R_{23}}-(-1)^p}{\sqrt{R_{23}}+(-1)^p}\right)^{1/2}} \quad (8.c)$$

In a particular case where the two couplers are identical, the equations become much simpler. Indeed, with $\theta_1 = \theta_2 = \theta$ and $\exp(-\alpha\Delta L) < 1$ condition, equations (6.c) and (6.d) can hold true if and only if $R_{14} = R_{23}$ and $n = p = 1$. Therefore, equations (8) become:

$$\alpha = -\frac{1}{\Delta L}\ln\left(\frac{\sqrt{R_{14}}-1}{\sqrt{R_{14}}+1}\right) = -\frac{1}{2\Delta L}\ln\left(\frac{\sqrt{R_{13}}+(-1)^l}{\sqrt{R_{13}}-(-1)^l}\cdot\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\right) \quad (9.a)$$

$$\kappa = \sin^2\theta = \frac{\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\right)^{1/2}}{1+\left(\frac{\sqrt{R_{13}}-(-1)^l}{\sqrt{R_{13}}+(-1)^l}\frac{\sqrt{R_{24}}+(-1)^m}{\sqrt{R_{24}}-(-1)^m}\right)^{1/2}} \quad (9.b)$$

In short, we have shown that it is possible to extract the values of the propagation loss constant α of the waveguide as well as the power coupling coefficients of the two couplers (κ_1 and κ_2) based on the four directly measurable extinction ratios R_{13}, R_{14}, R_{23} and R_{24}.

III. Experiment and Results

In this section we will describe the experiment implementation to carry out our measurement method and show several measured results based on the theory developed above.

As shown in Fig. 2, two experimental configurations can be used to record the optical spectra of transmissions through the UMZI structure. In our work, we used the configuration shown in Fig. 2.a with the Agilent tunable laser source (81640A) and

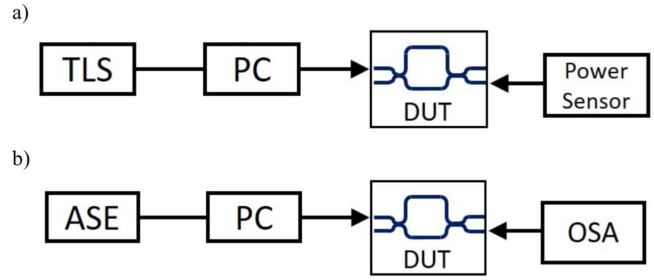

Fig. 2. a) The measurement setup using a tunable laser source (TLS), a single mode fiber polarization controller (PC) and a power sensor. b) An alternative configuration using a broadband source (*e.g.* ASE) with an optical spectrum analyzer (OSA).

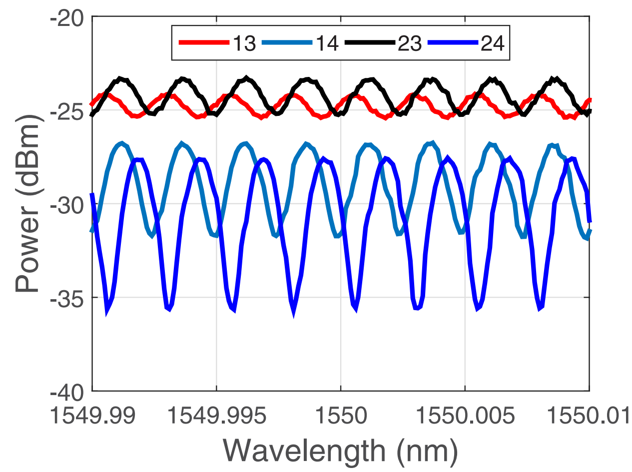

Fig. 3. Optical spectra of the transmissions through the UMZI fabricated on Si_3N_4 waveguide platform. The path length difference is 62.1 cm. The peaks and valleys do not line-up due to the limited ±1 pm of swept wavelength repeatability.

TABLE I
EXTINCTION RATIOS AND VALUES OF PARAMETERS *l, m, n, p*

R_{13} (dB)	R_{14} (dB)	R_{23} (dB)	R_{24} (dB)	*l*	*m*	*n*	*p*
1.20	4.93	1.86	8.00	1	1	1	1

power sensor (81635A) which can resolve a 0.1 pm step size in the optical spectrum. We will demonstrate the proposed method with the two following test cases:

a) A UMZI on Si_3N_4 waveguides with a 62.1 cm path length difference and two different directional couplers with unknown coupling ratios: We will use the general equation set (8) to calculate the waveguide propagation loss as well as the coupling ratios of the two couplers.

b) A UMZI on Si waveguides with 150 μm path length difference and two identical directional couplers: We will characterize the relation between the coupling ratio and the coupler length by applying simplified equation set (9) to multiple UMZIs with varying directional coupler lengths.

A. UMZI Formed by Two Different Directional Couplers

This is the most general case where all of the criteria are unknown. The transmitted optical spectra around 1550 nm wavelength are shown in Fig. 3. Values of the extinction ratios are extracted and tabulated in Table. 1. The extinction ratios

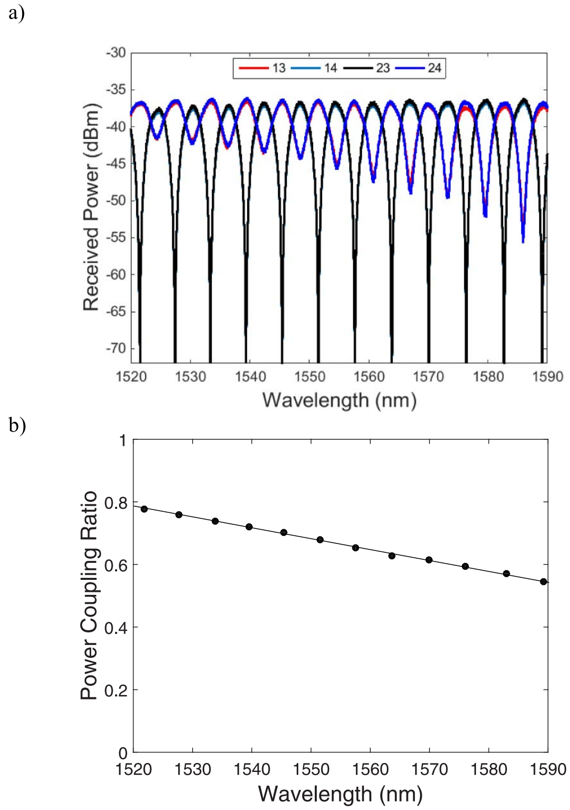

Fig. 4. a) Optical spectra of the transmissions through the UMZI fabricated on Si waveguides. The path length difference is 150 μm. b) Coupling coefficients of the directional coupler calculated from the transmission spectra shown in 4.a.

are converted to linear scale before we apply condition (7) to find out the relevant set of (*l, m, n, p*), which turns out to be (1, 1, 1, 1). With all of these parameters determined, by using Eq. (8), the propagation loss constant is calculated to be $\alpha = 0.0283 \pm 0.003$ cm^{-1}. This translates to a waveguide propagation loss rate of $\alpha_{dB} = 0.2458 \pm 0.0026$ dB/cm. The coupling ratios of the first and the second couplers are $\kappa_1 = 0.608 \pm 0.012$ and $\kappa_2 = 0.801 \pm 0.008$ respectively.

The propagation loss result by our method is in good agreement with the other method using optical frequency domain reflectometry (OBR 4400) [9], which measured the loss to be 0.24 dB/cm.

B. UMZI Formed by Two Identical Directional Couplers

In this particular test, we focus only on characterization of the coupling coefficients of the couplers. An array of the UMZI with varying coupler lengths (from 10 μm to 160 μm) are needed to characterize the coupler length dependence of the coupling coefficient. Therefore, a relatively short path length difference of 150 μm is made to save the space. This gives us a negligible propagation loss.

The transmission spectra of a UMZI with a coupler length of 80 μm is shown in Fig. 4.a. Using the simplified equations (9), we extracted the coupling coefficient over the broad wavelength range (Fig. 4.b). Repeating the process for other UMZIs with different coupler lengths, we can obtain

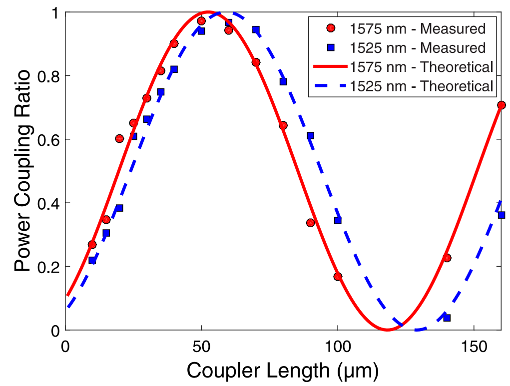

Fig. 5. The coupling ratio versus coupler length of the Si directional couplers at two wavelengths 1525 nm and 1575 nm. The measured data are plotted together with theoretical fitting curves.

the coupler length dependency of the coupling coefficients as shown in Fig. 5. The excellent agreement between the experimental results with theoretical calculation shows that this method is highly accurate.

IV. Conclusions

We analyzed the responses of unbalanced MZI structures formed by two 2×2 couplers. Based on this analysis, we have proposed and demonstrated a new method to simultaneously characterize the waveguide and the couplers using only a single test structure. The method is inherently robust to variations that can occur during sample preparation and testing, such as coupling loss, waveguide facet quality, and fiber-to-chip alignment. Precise results can be achieved in the presence of coupling variations.

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References

[1] D. B. Keck and A. R. Tynes, "Spectral response of low-loss optical waveguides," *Appl. Opt.*, vol. 11, no. 7, pp. 1502–1506, 1972.

[2] S. Taebi, M. Khorasaninejad, and S. S. Saini, "Modified Fabry–Perot interferometric method for waveguide loss measurement," *Appl. Opt.*, vol. 47, no. 35, pp. 6625–6630, 2008.

[3] P. Dumon *et al.*, "Low-loss SOI photonic wires and ring resonators fabricated with deep UV lithography," *IEEE Photon. Technol. Lett.*, vol. 16, no. 5, pp. 1328–1330, May 2004.

[4] T. R. Bourk, M. M. Z. Kharadly, and J. E. Lewis, "Measurement of waveguide attenuation by resonance methods," *Electron. Lett.*, vol. 4, no. 13, pp. 267–268, 1968.

[5] R. Adar *et al.*, "Measurement of very low-loss silica on silicon waveguides with a ring resonator," *Appl. Phys. Lett.*, vol. 58, no. 5, p. 444, 1991.

[6] M. Moresco *et al.*, "Method for characterization of Si waveguide propagation loss," *Opt. Exp.*, vol. 21, no. 5, pp. 5391–5400, 2013.

[7] W. Bogaerts *et al.*, "Silicon microring resonators," *Laser Photon. Rev.*, vol. 6, no. 1, pp. 47–73, Jan. 2012.

[8] A. M. Gutierrez, A. Brimont, M. Aamer, and P. Sanchis, "Method for measuring waveguide propagation losses by means of a Mach–Zehnder interferometer structure," *Opt. Commun.*, vol. 285, no. 6, pp. 1144–1147, Mar. 2012.

[9] J. Bauters *et al.*, "Planar waveguides with less than 0.1 dB/m propagation loss fabricated with wafer bonding," *Opt. Exp.*, vol. 19, no. 24, pp. 24090–24101, 2011.