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## Calibration-free measurement of high-speed Mach–Zehnder modulator based on low-frequency detection

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A calibration-free electrical method is demonstrated for measuring the frequency response of high-speed Mach-Zehnder modulators (MZMs) based on low-frequency detection. The method achieves the high-frequency modulation index and half-wave voltage measurement of MZMs by the low-frequency electrical spectrum analysis of the two-tone and bias-modulated optical signal. Moreover, it eliminates the need for correcting the responsivity fluctuation in the photodetector through setting a specific frequency relationship between the two-tone and bias modulation. Both absolute and relative frequency response of MZMs are experimentally measured with our method and compared with those obtained with conventional methods to check for consistency. © 2016 Optical Society of America

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The frequency response of Mach-Zehnder modulators (MZMs), including the modulation index and half-wave voltage, is critical to device characterization and system optimization especially for the wideband electrical-to-optical signal conversion [1,2]. Several methods have been demonstrated for measuring the frequency responses of high-speed modulators based on optical or electrical spectrum analysis [3-12]. The optical spectrum method was proposed for measuring modulation index and half-wave voltage of MZMs simply with a microwave source and an optical spectrum analyzer (OSA) [3-7]. The spectral resolution is on the order of 1 GHz (0.01 nm at 1550 nm) limited by the diffraction-grating-based OSA [3,4,7]. It could be improved to be tens of MHz with a Brillouin-based OSA or heterodyne-based OSA [13-16]. However, the optical spectrum analysis is often affected by linewidth of the laser source [17] because the modulated optical spectra are dependent on both the laser source and modulator.

The electrical swept frequency method is widely used for measuring the relative frequency response with a vector network analyzer (VNA) [10-12]. This method requires a wideband photodetector (PD) to cascade with the MZM under test, and it relies on de-embedding the contribution of the PD because the measured results depend on both the MZM and the PD [10]. An improved swept-frequency method was demonstrated for simplified calibration assisted by an electroabsorption modulator (EAM), in which the same EAM was used as the modulator or PD with almost identical frequency response [12]. Nevertheless, the major difficulty for the VNAbased swept-frequency methods when the absolute frequency response is involved is the required extra calibration of the PD responsivity [18,19]. As we know, the modulation index and half-wave voltage represent the absolute frequency response of MZMs, which are more informative than the relative frequency response because they reflect not only the relative change of modulation efficiency but also the absolute modulation efficiency itself. Thus, the methods that enable high-resolution modulation index and half-wave voltage measurement and meanwhile avoid any calibration for the responsivity of PD are of great importance.

In this Letter, we propose, for the first time to our knowledge, a calibration-free electrical method for measuring the modulation index and half-wave voltage of high-speed MZMs based on low-frequency detection. Our method consists of two-tone and bias modulation on the MZM under test. The two-tone and bias-modulated sidebands heterodyne with each other and generate the desired low-frequency beat notes after photodetection, which allows extracting the modulation index and half-wave voltage at the two-tone modulation frequency. Our method avoids any roll-off calibration for the PD responsivity and the bias drift of MZM because they can be canceled out by carefully choosing the frequency relationship between the two-tone and bias modulation. Moreover, the proposed method enables a low-speed PD and a low-frequency electrical spectrum analyzer (ESA) for measuring high-speed MZMs.

As shown in Fig. 1, an optical carrier is sent to the MZM under test onto which two closely spaced sinusoidal tones



**Fig. 1.** Schematic setup of our method. LD, laser diode; MS, microwave source; PC, polarization controller; MZM, Mach–Zehnder modulator; LFSG, low-frequency signal generator; LFPD, low-frequency photodetector; ESA, electrical spectrum analyzer.

 $v_1(t) = V_1 \sin(\omega_1 t + \varphi_1)$  and  $v_2(t) = V_2 \sin(\omega_2 t + \varphi_2)$ are applied. The two-tone modulated optical field can be expressed by [3,5]

$$E(t) = A_0 e^{j\omega_0 t} [1 + \gamma e^{jm_1 \sin(\omega_1 t + \varphi_1) + jm_2 \sin(\omega_2 t + \varphi_2) + j\varphi}], \quad (1)$$

where  $\omega_0$  and  $A_0$  are the angular frequency and amplitude of the optical carrier, respectively.  $\gamma$  and  $\varphi$  represent the asymmetric factor and phase bias of the MZM. The modulation index *m* of the MZM can be related to the half-wave voltage  $V_{\pi}$  by [4]

$$m_i = \pi V_i / V_{\pi}(\omega_i) = \pi \sqrt{2p_i z_L} / V_{\pi}(\omega_i)$$
 (*i* = 1, 2), (2)

with the microwave driving power  $p_i$  and the input impedance  $z_L$  of the MZM at  $\omega_i$ . As the two-tone frequencies are very close to each other, the half-wave voltages at these two frequencies can be considered to be the same  $(V_{\pi}(\omega_1) \approx V_{\pi}(\omega_2))$ ; thus, we have from Eq. (2):

$$a = m_2/m_1 = V_2/V_1.$$
 (3)

For a low-frequency detection, a low-frequency bias modulation is also applied through the bias port of the MZM, which is given by

$$\varphi = m_b \sin(\omega_b t + \varphi_b) + \varphi_{b0}, \qquad (4)$$

with the static phase bias  $\varphi_{b0}$ . The output optical signal of the MZM is detected by the PD to generate a photocurrent given by

$$i(t) = R|E(t)|^{2} = RA_{0}^{2}(1 + \gamma^{2}) + 2\gamma RA_{0}^{2}$$
  

$$\cdot \cos[m_{1} \sin(\omega_{1}t + \varphi_{1}) + m_{2} \sin(\omega_{2}t + \varphi_{2}) + m_{b} \sin(\omega_{b}t + \varphi_{b}) + \varphi_{b0}],$$
(5)

with responsivity R of the PD. Applying the Jacobi-Anger expansion [20,21] to Eq. (5), we have

For a calibration-free measurement with low-frequency detection, the two-tone frequencies are set very close to each other  $(\omega_1 \approx \omega_2 \gg \omega_b)$ , and their frequency difference is set about twice the frequency of the bias modulation  $(\omega_1 - \omega_2 \approx 2\omega_b)$  or close to dc  $(\omega_1 - \omega_2 \approx 0)$ . Therefore, it is reasonable to have the assumption on the responsivity  $R(\omega_1 - \omega_2 - \omega_b) \approx R(\omega_b)$  or  $R(\omega_1 - \omega_2 + \omega_b) \approx R(\omega_b)$  stands. In this case, we define the heterodyne ratio H based on Eqs. (7a) and (7b) as

$$H(m_i) = \left| \frac{i(\omega_1 - \omega_2 \pm \omega_b)}{i(\omega_b)} \right|$$
  
=  $\frac{J_1(m_1)J_1(m_2)}{J_0(m_1)J_0(m_2)}$ , (*i* = 1, 2). (8)

Equations (3) and (8) indicate the modulation index can be determined by detecting the desired low-frequency components of the two-tone and bias-modulated optical signal. As the responsivity R of PD is canceled out by carefully choosing the two-tone frequencies, our method realizes a calibration-free measurement of high-speed MZMs with low-frequency detection. Moreover, our method is bias-drift-free because it is insensitive to the static phase bias  $\varphi_{b0}$ . In practice, it will benefit from relatively high microwave power and a near quadrature bias phase due to the improved signal-to-noise ratio. Besides, Eqs. (7) and (8) are established without the small-signal assumption; thus, our method is applicable for different driving levels. It is worth noting that the phase  $\varphi_1$  and  $\varphi_2$  of the two-tone signals will not affect the amplitude of the desired low-frequency components; thus, it is not necessary to keep the two-tone signals synchronized, which makes for a simpler measurement.

In the experiment, the optical carrier at 1550 nm is sent to a LiNbO<sub>3</sub> MZM (AVANEX SD40), which is modulated by a 1.1 MHz spaced two-tone signal ( $f_1$  and  $f_2$ ) through its RF port and a 500 kHz sinusoidal signal ( $f_b$ ) through its bias port. The two-tone signal is generated from two microwave sources (HP 8340B, R&S SMP 04) and one power combiner (Wiltron K240B). The modulated optical signal is collected and analyzed by a low-frequency PD (with a bandwidth of 1 GHz) and ESA, which is also simultaneously monitored by an OSA (YOKOGAWA AQ6370C) for the subsequent experiment.

Figure 2 shows the electrical spectra of the modulated optical signal at several typical two-tone frequencies, respectively. For example, the desired frequency components are measured to be -6.35 dBm at 500 kHz ( $f_1 - f_2 - f_b$ ) and -58.37 dBm at 600 kHz ( $f_b$ ), respectively, in the case of  $f_1 = 24.06$  GHz,  $f_2 = 24.0589$  GHz and  $f_b = 500$  kHz. The actual microwave amplitudes of the two-tone signals applied on the

$$i(t) = RA_0^2 \left\{ \begin{array}{l} 1 + \gamma^2 + 2\gamma \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_p(m_1) J_q(m_2) J_k(m_b) \\ \cdot \cos(p\omega_1 t + p\varphi_1 + q\omega_2 t + q\varphi_2 + k\omega_b t + k\varphi_b + \varphi_{b0}) \end{array} \right\},\tag{6}$$

with the *p*th, *q*th, or *k*th-order Bessel function of the first kind  $J_p(\cdot)$ ,  $J_q(\cdot)$ , and  $J_k(\cdot)$ , respectively. It is easy to quantify the desired low-frequency components as the following:

$$i(\omega_b) = -4A_0^2 \gamma R(\omega_b) J_0(m_1) J_0(m_2) J_1(m_b) \sin \varphi_{b0}$$
 (7a)

and

$$i(\omega_1 - \omega_2 \pm \omega_b) = \pm 4A_0^2 \gamma R$$
  
(\omega\_1 - \omega\_2 \pm \omega\_b) J\_1(m\_1) J\_1(m\_2) J\_1(m\_b) \sin \varphi\_{b0}. (7b)

MZM are measured to be 0.252 V (-1.97 dBm) at  $f_1 =$  24.06 GHz and 0.233 V (-2.65 dBm) at  $f_2 =$  24.0589 GHz. Therefore, the modulation index of MZM is solved to be 0.104 ( $m_1$ ) at 24.06 GHz and 0.0961 ( $m_2$ ) at 24.0589 GHz, respectively. The half-wave voltage is determined to be 7.609 V at about 24.06 GHz. Note that the modulation index at 24.06 GHz is extracted from the two frequency components at 500 and 600 kHz, verifying the low-frequency detection of our method.



**Fig. 2.** Measured electrical spectra of the heterodyne signals in the case of different modulation frequencies.

The measurement can be easily operated at other frequencies  $f_1$  by simply setting  $f_2 = f_1 - 0.0011$  GHz. The electrical spectrum shows extremely narrow spectrum lines, which are independent of the linewidth of optical source because the modulations are operated on the same optical carrier [18,20]. For an automatic swept measurement, both the MS and the ESA are connected and controlled by a computer through a GPIB data bus, with which the two-tone frequencies are set, and the spectrum data from ESA are acquired by a MATLAB program. Figure 3 shows the frequency-dependent modulation index determined based on Eqs. (3) and (8). The actual microwave driving powers on the MZM are also given in Fig. 3 for reference, from which the half-wave voltage can be determined at different modulation frequencies, as illustrated in Fig. 4.

The modulation index is also measured using the conventional optical spectrum analysis under the same driving level. The measured modulation indices and half-wave voltage with the OSA method are also shown in Figs. 3 and 4, where the minimum measurable frequency of about 4 GHz is limited by the resolution of the OSA [4,17,20]. Nevertheless, our results are in good agreement with the data obtained using the conventional OSA method, verifying the calibration-free measurement of our method because the OSA method works without any photodetection.

For further verification, the MZM is measured with the VNA method. After a full two-port electrical-to-electrical calibration, the VNA (Agilent N5235A) is calibrated to two microwave coaxial ports, and the relative frequency response of the MZM cascaded with a broadband PD (HP 11982A) is obtained. The relative frequency response of PD is obtained



Fig. 3. Measured modulation indices versus modulation frequency, where the microwave driving powers are shown for reference.



Fig. 4. Measured half-wave voltage versus modulation frequency with our method (red line) and OSA method (blue open circles).

with the optical frequency-detuned heterodyne method [16]; then, the relative frequency response of the modulator itself can be calibrated. The relative frequency responses of the MZM without and with calibration are also illustrated in Fig. 5 for comparison, where the consistency also proves our calibration-free measurement. It is noteworthy that the relative frequency response can be easily obtained from the half-wave voltage, but not vice versa, due to the requirement of the absolute responsivity Rof PD. In contrast, our method easily provides the modulation index, the half-wave voltage, and the relative frequency response with the calibration-free measurement.

For completeness, we investigate the error dependence of the measured modulation index on the uncertainty of heterodyning ratio H and the amplitude ratio a by the error transfer factors as the following:

$$\delta m_1/m_1 = F_1 \cdot \delta H/H + F_2 \cdot \delta a/a, \qquad (9a)$$

with

$$F_{1} = (\delta m_{1}/m_{1})/(\delta H/H)$$

$$= \frac{1}{m_{1} \left[ \frac{f_{1}(m_{1})}{f_{0}(m_{1})} + \frac{f_{0}(m_{1}) - f_{2}(m_{1})}{2f_{1}(m_{1})} + \frac{af_{1}(am_{1})}{f_{0}(am_{1})} + \frac{af_{0}(am_{1}) - af_{2}(am_{1})}{2f_{1}(am_{1})} \right]},$$
(9b)

and



**Fig. 5.** Measured half-wave voltages (in dB) with our method (red line) and the relative frequency response with the VNA method (blue open squares, blue open circles).

$$F_{2} = (0m_{1}/m_{1})/(0d/d)$$

$$= \frac{\frac{J_{2}(am_{1}) - J_{0}(am_{1})}{J_{1}(am_{1})} - \frac{2J_{1}(am_{1})}{J_{0}(am_{1})}}{\frac{2J_{1}(m_{1})}{aJ_{0}(m_{1})} + \frac{J_{0}(m_{1}) - J_{2}(am_{1})}{aJ_{1}(m_{1})} + \frac{2J_{1}(am_{1})}{J_{0}(am_{1})} + \frac{J_{0}(am_{1}) - J_{2}(am_{1})}{J_{1}(am_{1})}},$$
(9c)

which is derived from the total derivative of Eq. (8). In our experiment, the power difference of the two-tone microwave signal is within 3 dB, corresponding to an amplitude ratio of 0.7 <a < 1.4. From Fig. 6, it is easily read that  $F_1 < 0.5$  and  $F_2 < 0.5$ 0.7 in the case of the modulation index  $m_1 < 0.3$ . In our measurement, the main error sources come from the response fluctuation of PD and the uncertainty of ESA because our method is based on the approximation of  $R(f_1 - f_2 - f_b) \approx R(f_b)$ . The response difference is estimated to be less than 0.1 dB within the frequency difference of 100 kHz according to the PD's specification. The ESA holds a power uncertainty of less than 0.1 dB or an amplitude uncertainty of less than 0.05 dB. Therefore, the measured heterodyning ratio H would have an uncertainty of less than 0.3 dB (=  $2 \times 0.1 + 0.1$ ), which means a relative error of less than 1.76% (= (0.5 \* (10(0.3/20) - 1) \* 100%) might be delivered to the extracted modulation index. Meanwhile, the measured amplitude ratio a has an uncertainty of less than 2.33% (0.2 dB in power), corresponding to an uncertainty of less than 1.63% (= 0.7 \* 2.33%) delivered to the modulation index. Besides, the assumption of  $V_{\pi}(f_1) \approx V_{\pi}(f_2)$  could affect the uncertainty of amplitude ratio a and henceforth the modulation index. The half-wave voltage difference is estimated to be less than 2% at the 1.1 MHz spaced two-tone frequencies, corresponding to an uncertainty of less than 1.4% (= 0.7 \* 2%) transferred to the modulation index. In total, the uncertainty of the measured modulation index is estimated to be less than 4.79% (= 1.76% + 1.63% + 1.4%) in the worst case. The contributed uncertainty from the assumption  $V_{\pi}(f_1) \approx$  $V_{\pi}(f_2)$  can be further reduced by choosing closer two-tone frequencies if the half-wave voltages vary sharply at a certain frequency range.

Furthermore, the uncertainty of the half-wave voltage can be similarly written by

$$\delta V_{\pi}/V_{\pi} = \delta V_1/V_1 - \delta m_1/m_1.$$
<sup>(10)</sup>

Because the measured electrical power has an uncertainty of less than 0.1 dB (1.16% = (10.(0.1/20) - 1) \* 100%), the total relative error of less than 5.95% (= 1.16% + 4.79%) might be delivered to the measured half-wave voltage in the worst case.



**Fig. 6.** Error transfer factors as a function of modulation index in different cases of amplitude ratio a of the two-tone signal.

Compared with the OSA method, the proposed electrical method achieves a high-resolution measurement and, at the same time, avoids the linewidth influence of the laser source due to the two-tone and bias modulations on the same optical carrier. In contrast with the VNA method, our method not only realizes the absolute frequency response measurement but also eliminates the roll-off and correction of PD responsivity at the cost of an addition microwave source. Different from other electrical methods, our method enables the calibration-free, bias-drift-free, and PD roll-off-free measurement of modulation index and half-wave voltage with low-frequency detection in spite of the involved photodetection.

In summary, we have demonstrated an electrical method for characterizing the modulation index of MZMs based on lowfrequency detection. Our method provides a high-resolution absolute frequency response measurement and eliminates the need for correcting the responsivity fluctuation of PD, leading to calibration-free and bias-drift-free electrical measurement for high-speed MZMs with low-frequency detection.

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