Solenoidal Heat-Flux in Quasi-Ballistic Thermal Conduction

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The goal

- Goal: *Recast the Boltzmann transport equation (BTE) into an enhanced Fourier law for accurate device thermal simulation outside Fourier law* [1]
  - Fourier law: $\boldsymbol{q} = -\kappa \nabla T$
  - Enhanced Fourier law:

$$\boldsymbol{q} = -\kappa \nabla T + \frac{3}{5}\kappa^{HF}(\Lambda^{LF})^2 \nabla(\nabla^2 T) - \frac{1}{5}(\Lambda^{LF})^2 \nabla \times (\nabla \times \boldsymbol{q}) + \frac{3}{5}(\Lambda^{LF})^2 \nabla(\nabla \cdot \boldsymbol{q})$$

- So what's new here compared to [2]?
  - New formulation - entirely in terms of total heat-flux, and reservoir temperature

- Derived from the BTE - not a phenomenological model

[1] A. T. Ramu and J. E. Bowers, *J. Appl. Phys.* 118, 125106 (2015)
[2] G. Chen, *Physical Review Letters* 86, no. 11 (2001): 2297

Solenoidal heat-flux

- Identified new term in constitutive relation
- Fourier law heat-flux is curl-free
- Quasi-ballistic transport involves a divergence-free, solenoidal ('curly') term!
- Derivation from the BTE:

[1]A. T. Ramu and J. E. Bowers, *J. Appl. Phys.* 118, 125106 (2015)

Solenoidal heat-flux

$$\boldsymbol{q} = -\frac{1}{5}(\Lambda^{LF})^2 \nabla \times (\nabla \times \boldsymbol{q}) + \frac{3}{5}(\Lambda^{LF})^2 \nabla(\nabla \cdot \boldsymbol{q}) - \kappa \nabla T - \frac{3}{5}\kappa^{HF}(\Lambda^{LF})^2 \nabla(\nabla^2 T)$$

Derivation from the BTE: A. T. Ramu and J. E. Bowers, *J. Appl. Phys.* 118, 125106 (2015)

$\boldsymbol{q}$**=** Net heat flux in both LF and HF channels

Λ^{LF}=Mean-Free Path (MFP) of quasi-ballistic LF modes

κ=bulk thermal conductivity

κ^{HF}=reservoir (HF) mode thermal conductivity

T=Temperature of HF channel

Solenoidal heat-flux

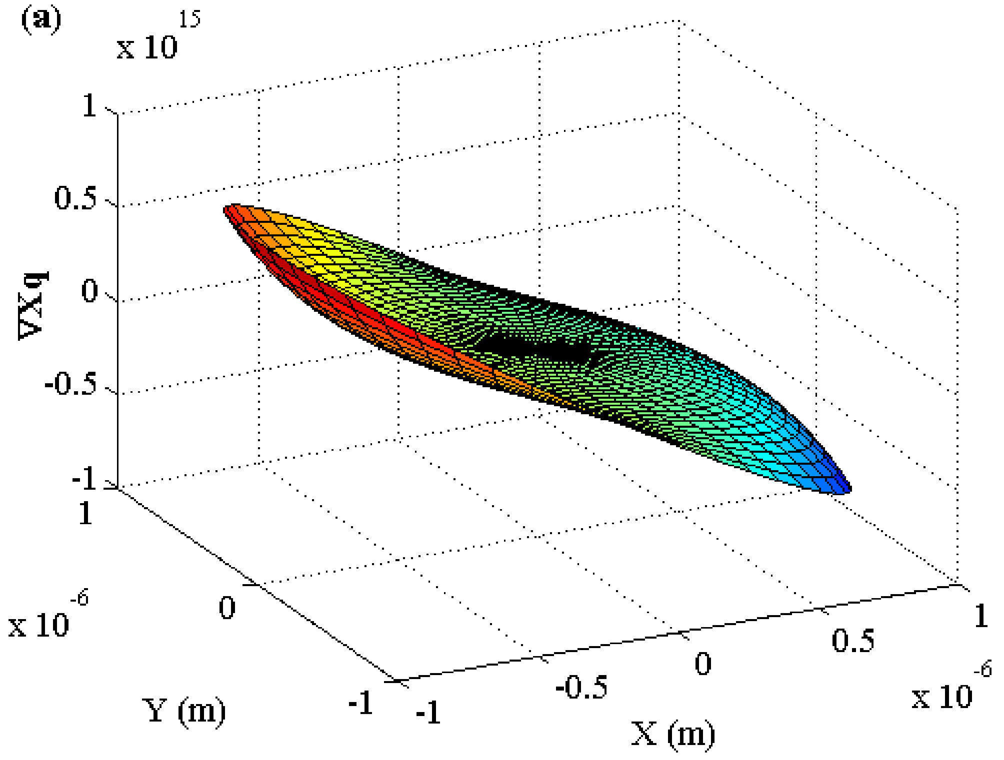
$$\boldsymbol{q} = -\frac{1}{5}(\Lambda^{LF})^2 \nabla \times (\nabla \times \boldsymbol{q}) + \frac{3}{5}(\Lambda^{LF})^2 \nabla(\nabla \cdot \boldsymbol{q}) - \kappa \nabla T - \frac{3}{5}\kappa^{HF}(\Lambda^{LF})^2 \nabla(\nabla^2 T)$$

Derivation from the BTE: A. T. Ramu and J. E. Bowers, *J. Appl. Phys.* 118, 125106 (2015)

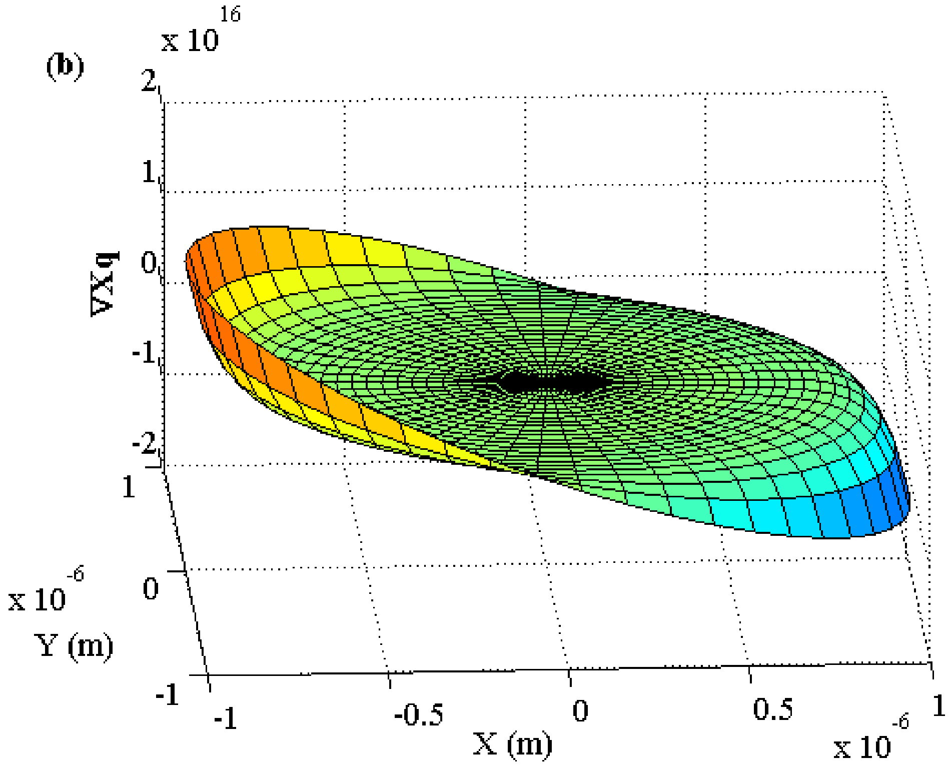
- Applied to heat transport in a cylinder
- Both temperature and heat-flux needed on cylinder periphery
- Extra boundary conditions are the consequence of two-channel model and dropped terms
- ‘Curly’ (solenoidal) heat-flux observed in the quasi-ballistic regime

Solenoidal heat-flux

BCs: $T(R,\theta) = T_0 sin\theta, \boldsymbol{q}(R,\theta)\cdot\hat{e}_r = -Q_0 sin\theta$



Λ^{LF}=0.6 micron



Λ^{LF}=0.15 micron

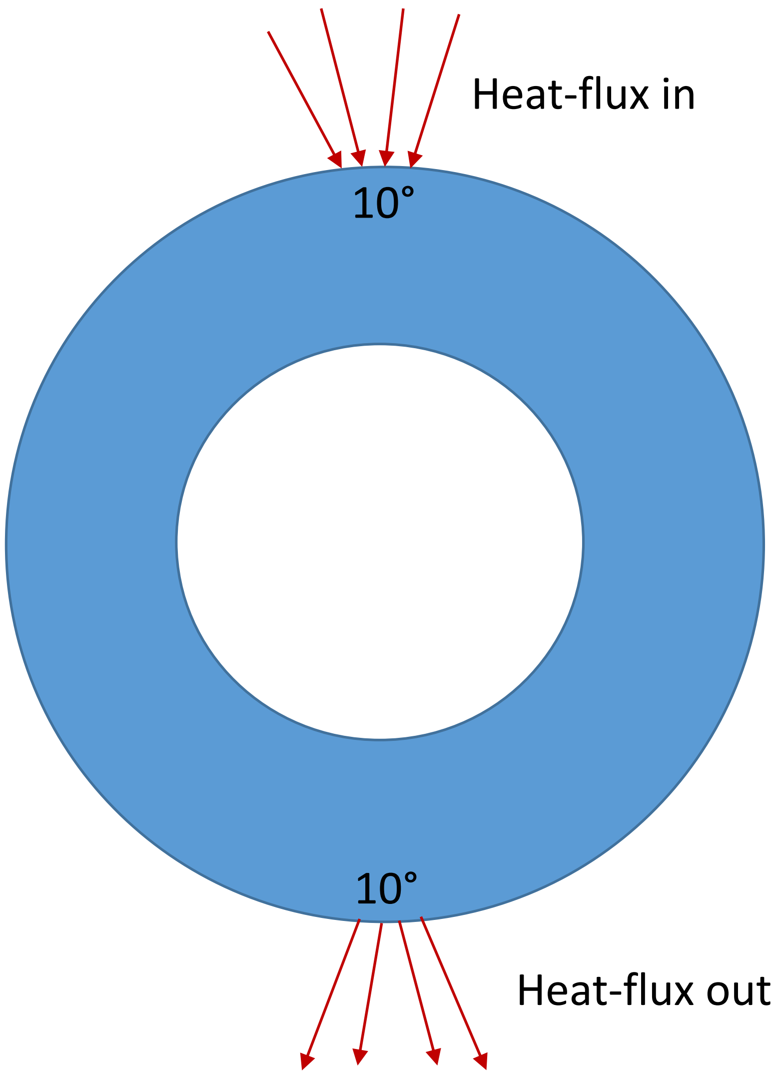
Quasi-ballistic transport is essential to the observation of the solenoidal heat-flux.

Applications

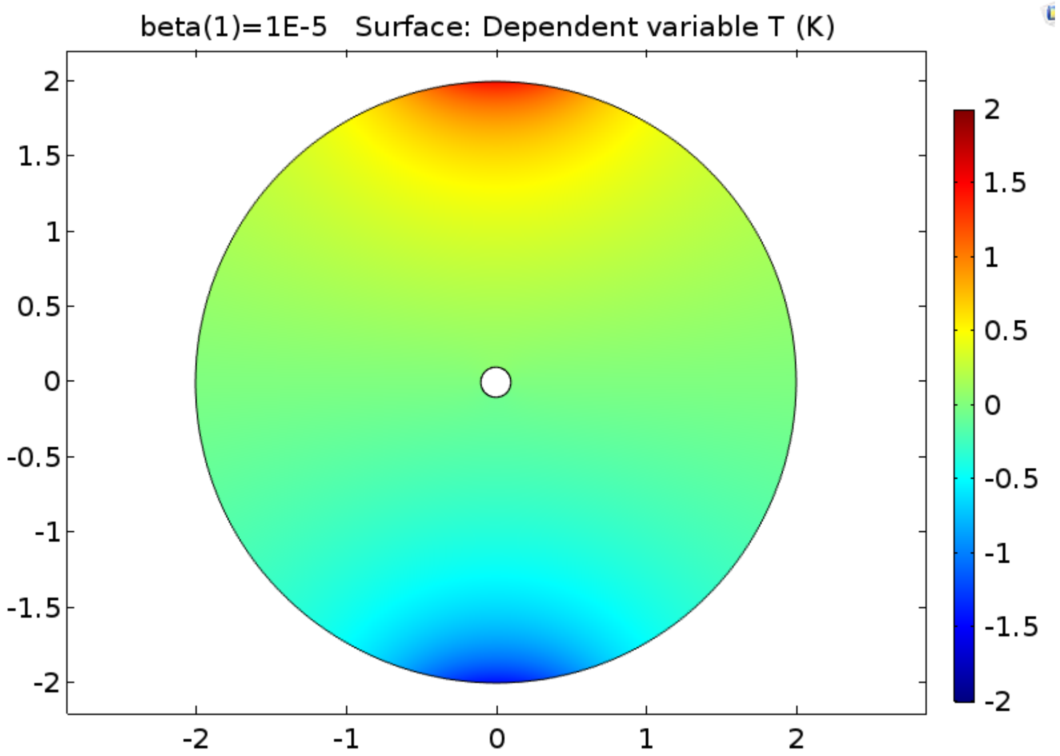
- Simultaneously confined phonons and optical modes
  - 10 GHz silicon phonon ring resonator
  - Phonon wavelength ~ 1 micron, Mean-free path ~ 10s of microns
  - Enhanced stimulated Brillouin scattering of light
- *Circulating heat fluxes reduce the effective thermal conductivity!*[3]
  - Circulating heat-flux fails to equilibrate with lattice at the cold end
  - Potentially of great importance for thermoelectric applications

[3] Ashok T. Ramu, Carl D. Meinhart and John E. Bowers, “Circulation of the heat-flux reduces the effective thermal conductivity” (under preparation, 2015)

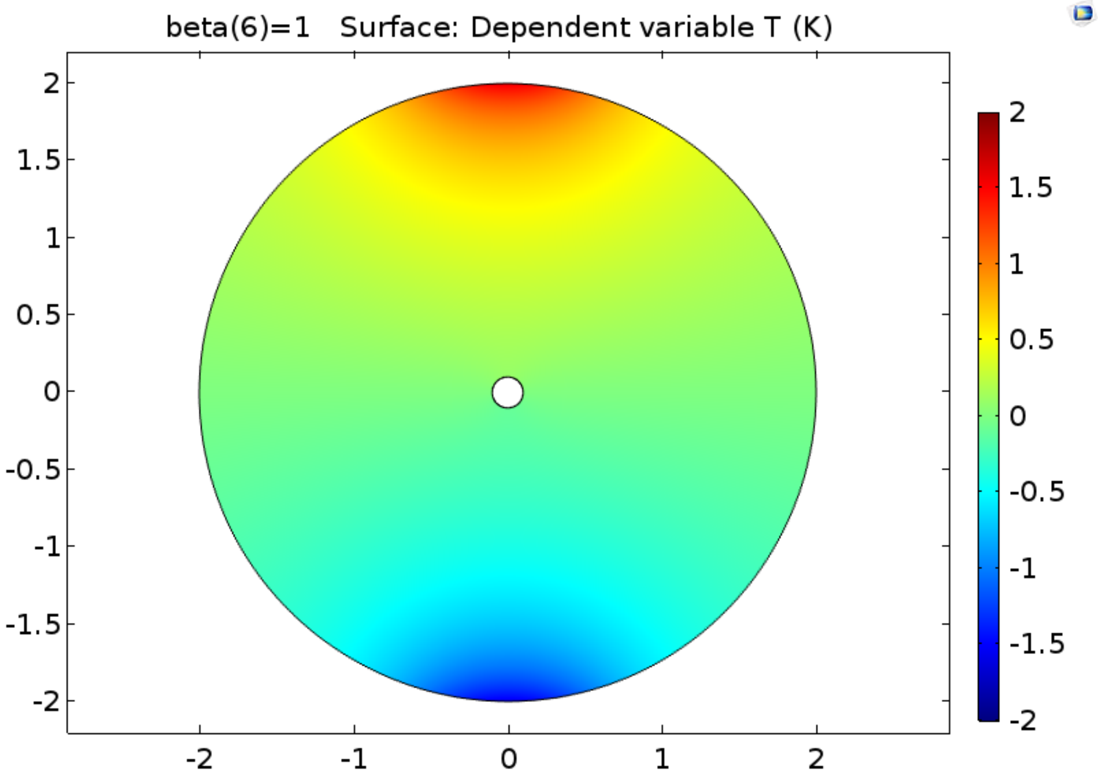
Reduction of effective thermal conductivity



Reduction of effective thermal conductivity



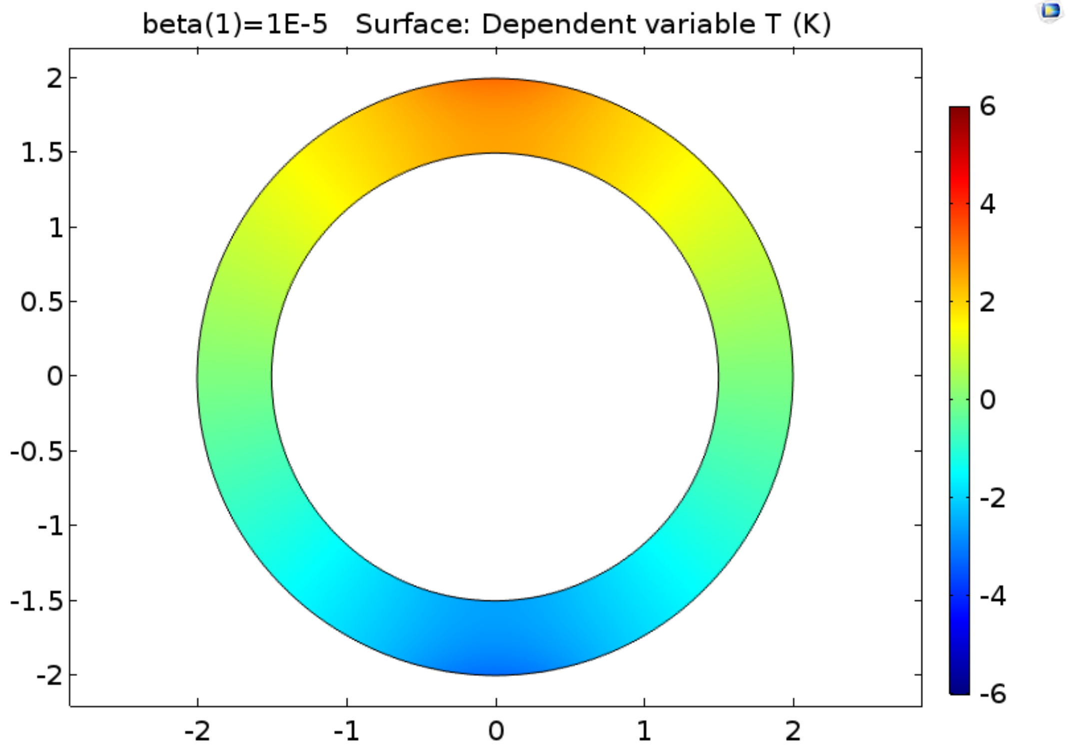
Circulation turned OFF: Hot side temperature = 300+1.45 K

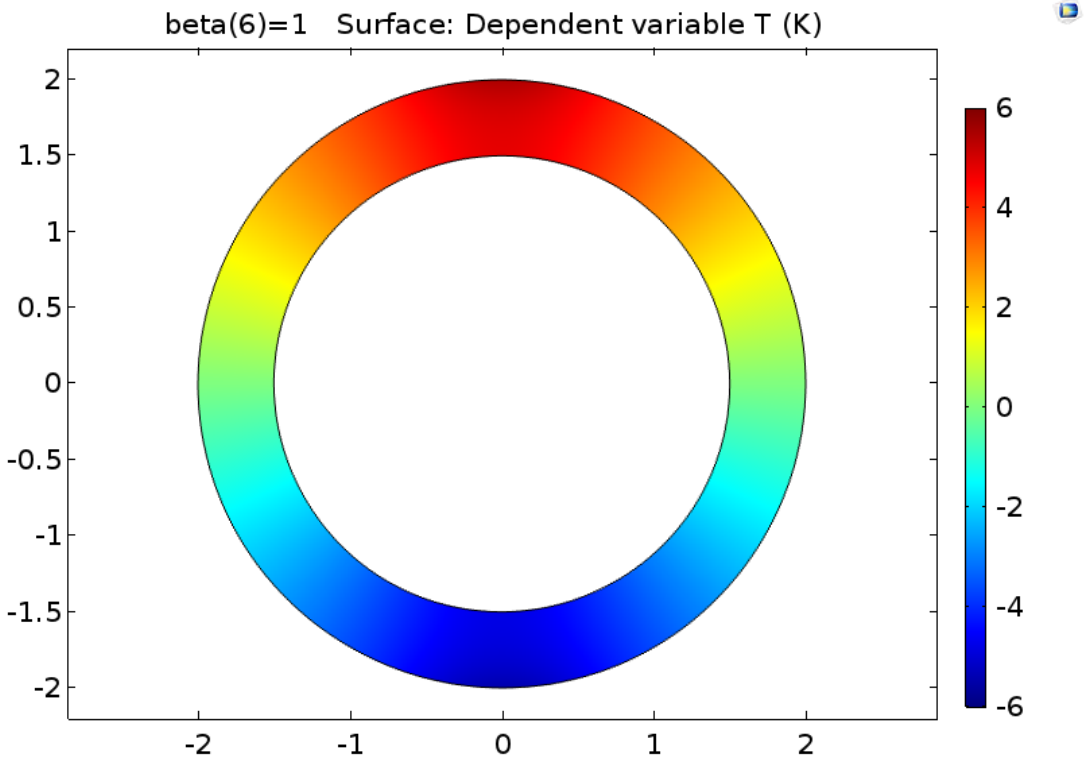


Circulation turned ON: Hot side temperature = 300+1.56 K

Annulus of outer diameter 2 micron, inner diameter 0.1 micron – negligible contribution from solenoidal term
LF mode thermal conductivity = 60 W/m-K; LF mode mean-free path = 500 nm.
HF mode thermal conductivity = 30 W/m-K

Reduction of effective thermal conductivity





Circulation turned OFF: Hot side temperature = 300+3.24 K

Circulation turned ON: Hot side temperature = 300+5.47 K

Annulus of outer diameter 2 micron, inner diameter 1.5 micron – LARGE CIRCULATORY EFFECT
LF mode thermal conductivity = 60 W/m-K; LF mode mean-free path = 500 nm.
HF mode thermal conductivity = 30 W/m-K

Summary

- A new circulatory term identified in the enhanced Fourier law
- ‘Curly’ (solenoidal) heat-flux observed numerically in the quasi-ballistic regime
- Circulating heat fluxes reduce the effective thermal conductivity

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Thank you for your time!

If you have any questions, please contact Dr. Ashok T. Ramu at ashok.ramu@gmail.com